Induced Lorentz and PCT Symmetry Breaking in an External Electromagnetic Field

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Abstract

In this work we derive the Lorentz-PCT-violating effective action for a fermion in a constant and uniform electromagnetic field using the Fock-Schwinger proper time method and extract the exact value of the coefficient of the nonperturbatively induced Chern-Simons term.

Lattely, there has been increasing interest in extensions of quantum electrodynamics (QED) and the Standard Model (SM), where Lorentz and PCT symmetries are broken in the fermion sector of the corresponding Lagrangian density, both from phenomenological[1−3] and field-theoretical standpoints[4−6].

If we consider radiative corrections, the Lorentz- and PCT-violating axial vector term induces a four dimensional analogue of the so-called Chern-Simons (CS) term in the free electromagnetic Lagrangian density of the extended Maxwell electrodynamics, in much the same way as an odd-parity mass term for fermions in QED in three-dimensional spacetime is responsible for the topological mass term of the gauge boson[7].

Nevertheless, in both cases, ambiguities occur in the usual perturbation theory, at leading order in the relevant expansion parameters: the coefficient of the induced CS term is regularization dependent and may only be fixed by physical requirements[8].

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In this work we address the issue of generation of a Lorentz- and PCT-violating CS term in the Lagrangian density of extended QED in (3+1) dimensions, by considering a fermion in the presence of a constant and uniform electromagnetic field. Employing the Fock-Schwinger proper time construct[9] together with the analytic regularization method[10] for the singular fermion Green's function, we derive the renormalized effective action corresponding to the CS term and extract its coefficient without any further approximations, confronting our result with those obtained from nonperturbative calculations in the sense of reference [6] and other recent approaches [11,12].

The effective action for a fermion in the presence of an external electromagnetic field in the extended version of QED considered in reference [1] is defined by

$$
\begin{aligned}
S_{\rm eff}(A) &= -i\log\int \mathcal{D}\overline{\psi}\mathcal{D}\psi\exp\left(i\int d^4x\,\overline{\psi}\;[i\partial\!\!\!/ - m - eA\!\!\!/(x) + b\!\!\!/\gamma_5]\;\psi\right)\\
&= \log\det\,[i\partial\!\!\!/ - m - eA\!\!\!/(x) + b\!\!\!/\gamma_5]\ \ , \qquad (1)
\end{aligned}
$$

where b_μ in the last term is a four-component *constant* quantity which picks out a preferred direction in spacetime. Such a term as it stands violates both Lorentz and PCT symmetry.

We are only interested in the CS sector, which is linear in $A_\mu(x)$ and arises from the induced vacuum polarization contribution to the total fermion current. This can be obtained according to the prescription[13]

$$
\frac{\delta S_{\rm eff}}{\delta A_\mu(x)} = \langle j^\mu(x)\rangle = ie\ {\rm tr}\left(\gamma_\mu \tilde{G}(x,x')\right)\Big|_{x=x'}\ \ , \qquad (2)
$$

where $\tilde{G}(x,x')$ is the gauge invariant part of the fermion Green's function $G(x,x')$, which satisfies the inhomogeneous differential equation

$$
[i\partial\!\!\!/ - m - eA\!\!\!/(x) + b\!\!\!/\gamma_5]\,G(x,x') = \delta^{(4)}(x-x')\ . \qquad (3)
$$

Now, we consider the Green's function

$$
\begin{aligned}
G(x,x') &= \langle x|[i\partial\!\!\!/ - m - eA\!\!\!/(x) + b\!\!\!/\gamma_5]^{-1}|x'\rangle\\
&= (i\partial\!\!\!/ + m - eA\!\!\!/(x) + b\!\!\!/\gamma_5)\,S(x,x')\ , \qquad (4)
\end{aligned}
$$

where $S(x,x')$ in momentum space satisfies the equation

$$
[ip\!\!\!/ - m - eA\!\!\!/(x) + b\!\!\!/\gamma_5][ip\!\!\!/ + m - eA\!\!\!/(x) + b\!\!\!/\gamma_5]\ S(p) = 1 \qquad (5)
$$

with $\pi_\mu = p_\mu - eA_\mu$. The above equation can still be rewritten as

$$[\hat{\pi}^2 - \frac{e}{2}\sigma.F - m^2 - 2b^2]\ S = 1\ , \tag{6}$$

where $\hat{\pi}_\mu = \pi_\mu - i\sigma_{\mu\nu}b^\nu\gamma_5$.

If we interpret the operator acting on S in equation (6) as the Hamiltonian of a quantum mechanical system, we can set the equations of motion

$$\begin{aligned} \dot{x}_\mu &= -i\,[x_\mu, H] \;=\; -2\hat{\pi}_\mu\ , \\ \dot{\hat{\pi}}_\mu &= -i\,[\hat{\pi}_\mu, H] \;=\; -2e\ F_{\mu\nu}\hat{\pi}^\nu\ , \end{aligned} \tag{7}$$

where the dotted operators mean derivatives with respect to a proper time parameter s. This system admits the solution

$$\hat{\pi}_\mu(s) = -\frac{1}{2}\left[\frac{eF\mathrm{e}^{-2eFs}}{\sinh(eFs)}\right]_{\mu\nu}\ \Delta x^\nu\ , \tag{8}$$

where $\Delta x^\nu = (x^\nu(s) - x^\nu(0))$. Hence, the Hamiltonian takes the form

$$\begin{aligned} H &= \Delta x^\mu K_{\mu\nu}\Delta x^\nu - 2x^\mu(s)K_{\mu\nu}x^\nu(0) - \frac{i}{2}\mathrm{tr}[eF\coth(eFs)] \\ &- \frac{e}{2}\sigma.F - m^2 - 2b^2\ , \end{aligned} \tag{9}$$

where $K_{\mu\nu} = \frac{1}{4}e^2F^2[\sinh^{-2}(eFs)]_{\mu\nu}$. Defining the evolution operator

$$U(x,x';s) = \langle x|\mathrm{e}^{-iHs}|x'\rangle\ , \tag{10}$$

we obtain

$$\begin{aligned} U(x,x';s) &= \frac{C(x,x')}{s^2}\exp\left\{-\frac{1}{2}\mathrm{tr}\log\left[(eFS)^{-1}\sinh(eFs)\right]\right\} \\ &\times\ \exp\left\{\frac{i}{4}\Delta x\,[eF\coth(eFs)]\,\Delta x + \frac{ie}{2}\sigma.Fs + i(m^2 - 2b^2)s\right\} \\ &= \langle x(s)|x(0)\rangle\ , \end{aligned} \tag{11}$$

where $\Delta x_\nu = (x_\nu - x'_\nu)$.

From equation (6), we have in coordinate representation

$$S(x,x') = \langle x|H^{-1}|x'\rangle\ . \tag{12}$$

This Green's function is singular in the ultraviolet (UV) limit $x \to x'$ and such singularity will manifest as an UV logarithmic divergence in the Lorentz- and PCT-violating sector of the effective action. In order to perform calculations leading to renormalized physical quantities, we adopt the analytic regularization scheme[10], with the replacement

$$\begin{aligned} S(x,x') \quad \longrightarrow \quad & S_\lambda(x,x') = f_\lambda\, m^{2\lambda} \langle x| \frac{1}{(H+i\epsilon)^{1+\lambda}} |x'\rangle \\ = \quad & \frac{f_\lambda\, m^{2\lambda}(-i)^{1+\lambda}}{\Gamma(1+\lambda)} \langle x| \int_0^\infty ds\, s^\lambda e^{is(H+i\epsilon)} |x'\rangle \\ = \quad & \frac{f_\lambda\, m^{2\lambda}(-i)^{1+\lambda}}{\Gamma(1+\lambda)} \int_0^\infty ds\, s^\lambda\, U(x,x';-s) e^{-s\epsilon} \ , \end{aligned} \tag{13}$$

where the parameter $\lambda > -1$ ultimately goes to zero to recover the original theory. In the above expressions f_λ is an arbitrary function of λ which is equal to the unity for $\lambda = 0$. The factor $m^{2\lambda}$ gives the correct dimension of the regularized propagator associated to the squared Hamiltonian H.

We also note that

$$(i\partial_\mu - eA_\mu(x)) \langle x(-s)|x(0)\rangle = \langle x|\pi_\mu(-s)|x'\rangle \ . \tag{14}$$

Then, it follows from (8) that $C(x,x')$ in (11) satisfies

$$\left(i\partial_\mu - eA_\mu(x) - \frac{e}{2} F_{\mu\nu}(x-x')^\nu - i\sigma_{\mu\nu}\, b^\nu \gamma_5 \right) C(x,x') = 0 \ , \tag{15}$$

whose solution is

$$C(x,x') = \mathcal{C}\Phi(x,x') \ , \tag{16}$$

where we have defined

$$\Phi(x,x') \equiv \exp\left(-ie \int_{x'}^{x} d\eta^\mu \ \left[A_\mu(\eta) + \frac{1}{2} F_{\mu\nu}(x-x')^\nu \right] + \Delta x^\mu \sigma_{\mu\nu}\, b^\nu \gamma_5 \right) \ . \tag{17}$$

From the normalization condition

$$\lim_{s\to 0} \langle x(s)|x'(0)\rangle = \lim_{s\to 0} \langle \tilde{x}(s)|\tilde{x}'(0)\rangle = \delta^{(4)}(x-x') \ , \tag{18}$$

we obtain

$$\mathcal{C} = -\frac{i}{(4\pi)^2} \ , \tag{19}$$

where $\tilde{x}_\mu = x_\mu + 2is\sigma_{\mu\nu}b^\nu\gamma_5$.

Thus, assuming that equation (4) holds for regularized Green's functions,

$$G_\lambda(x,x') = e^{ie\mathcal{P}\int_{x'}^{x} d\eta^\mu\, A_\mu(\eta)}\,\tilde{G}_\lambda(x,x')\ , \tag{20}$$

where

$$\begin{aligned}
\tilde{G}_\lambda(x,x') \;&=\; \frac{-f_\lambda\, m^{2\lambda}(-i)^{1+\lambda}}{\Gamma(1+\lambda)(4\pi)^2}\int_0^\infty ds\, s^{\lambda-2}\, e^{-is(m^2+b^2-i\epsilon)} \\
&\times\; \left\{\frac{1}{2}\Delta x^\mu\left(\left[eF\coth(eFs)\right]_{\mu\nu} - eF_{\mu\nu}\right)\gamma^\nu - 2\not{b}\gamma_5 + m\right\} \\
&\times\; \frac{e\sqrt{2\mathcal{F}}s}{\sin(e\sqrt{2\mathcal{F}}s)}\left[\cos(e\sqrt{2\mathcal{F}}s) - \frac{i\sigma.\,F}{\sqrt{2\mathcal{F}}}\sin(e\sqrt{2\mathcal{F}}s)\right] \\
&\times\; \exp\left\{-\frac{i}{4s}\Delta x_\mu\left[eFs\coth(eFs)\right]^{\mu\nu}\Delta x_\nu + \Delta x^\mu\sigma_{\mu\nu}b^\nu\gamma_5\right\}\ ,
\end{aligned} \tag{21}$$

with $\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$. This quantity appears together with

$$\mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}.\,\vec{B}$$

if we expand the exponential factor of the product $\sigma.\,F$ in equation (11). We may choose $\vec{E}\perp\vec{B}$ such that the first exponential in (11) also simplifies, leading to the above result.

Recalling equation (2), the expectation value of the regularized fermion current is found to be

$$\begin{aligned}
\langle j_\lambda{}^\mu\rangle \;&=\; ie\ \ \mathrm{tr}\left(\gamma_\mu\tilde{G}_\lambda(x,x')\right)\Big|_{x=x'} \\
&=\; \frac{-ief_\lambda\, m^{2\lambda}(-i)^{1+\lambda}}{\Gamma(1+\lambda)(4\pi)^2}\int_0^\infty ds\, s^{\lambda-2}\, e^{-is(m^2+b^2-i\epsilon)}\mathrm{tr}\left\{(-2b_\nu\gamma^\mu\gamma^\nu\gamma_5)\right. \\
&\quad\; \left.\frac{e\sqrt{2\mathcal{F}}s}{\sin(e\sqrt{2\mathcal{F}}s)}\left[\cos(e\sqrt{2\mathcal{F}}s) - \frac{i\sigma.\,F}{\sqrt{2\mathcal{F}}}\sin(e\sqrt{2\mathcal{F}}s)\right]\right\} \\
&=\; \frac{2e^2 f_\lambda\, m^{2\lambda}(-i)^{1+\lambda}}{\Gamma(1+\lambda)(4\pi)^2}\int_0^\infty ds\, s^{\lambda-1} b_\nu\ \mathrm{tr}(\gamma_5\gamma^\mu\gamma^\nu\sigma^{\alpha\beta})\, F_{\alpha\beta}\, e^{-is(m^2+b^2-i\epsilon)} \\
&=\; \frac{-ie^2 f_\lambda\, m^{2\lambda}(-i)^{1+\lambda}}{\Gamma(1+\lambda)\pi^2}\int_0^\infty ds\, s^{\lambda-1}\, e^{-is(m^2+b^2-i\epsilon)}\, b_\nu\tilde{F}^{\nu\mu}\ .
\end{aligned} \tag{22}$$

Therefore, the regularized CS effective action reads

$$S_\lambda^{CS} = a_\lambda\int d^4x\, b_\nu\tilde{F}^{\nu\mu}A_\mu\ , \tag{23}$$

where

$$\begin{aligned} a_\lambda &= \frac{-ie^2 f_\lambda\, m^{2\lambda}(-i)^{1+\lambda}}{\Gamma(1+\lambda)\pi^2} \int_0^\infty ds\, s^{\lambda-1}\, e^{-is(m^2+b^2-i\epsilon)} \\ &= \frac{-ie^2 f_\lambda\, m^{2\lambda}(-i)^{1+\lambda}}{\Gamma(1+\lambda)\pi^2} \frac{\Gamma(\lambda)}{i^\lambda (m^2+b^2-i\epsilon)^\lambda}\ . \end{aligned} \tag{24}$$

Expanding a_λ in Laurent series around $\lambda = 0$ and retaining the finite real part, we arrive at the renormalized CS effective action, with coefficient

$$a = \frac{e^2}{\pi^2}\left[\log\left(\frac{m^2}{m^2+b^2}\right) + f_0'\right]\ , \tag{25}$$

where f_0' stands for the derivative of f_λ at $\lambda = 0$. This completes our calculation.

We have performed a nonperturbative calculation of the coefficient of the CS contribution to the gauge-invariant effective action of a fermion in a constant uniform electromagnetic field. From (25), we see that the CS coefficient a exhibit a logarithmic contribution, which is *analytic* in b^2, but singular for $m \to 0$, in contrast with the opposite behavior of the corresponding contribution found in reference [12], which is non-analytic in b^2 and vanishes for $m \to 0$. The later situation is expected neither in perturbation theory[4], at leading order in b_μ, nor in the lowest order approximation considered in [6] and [11].

In our case, for $b^2 << m^2$ we have

$$a \sim \frac{e^2}{\pi^2}\left[-\frac{b^2}{m^2} + f_0'\right]\ .$$

The first term in the last expression comes from the singular sector of the CS effective action and is absent from the finite lowest order result derived in reference [6]. However, the gauge-invariant vacuum polarization amplitude is logarithmically divergent and must be regularized *as a whole* object. This would lead to the above mentioned arbitrariness in the coefficient of the CS term. The same reasoning applies to the calculation performed in reference [11].

The freedom of choice of f_λ in the analytic regularization scheme we have adopted is reflected by the presence of a residual contribution proportional to f_0'. This arbitrary constant is not determined from the symmetries of the theory and can only be fixed through subsidiary physical constraints set by experiment[1],[3].

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