The $SU(2)$ Skyrme model and anomaly*

E. M. C. Abreu [a†], J. Ananias Neto and W. Oliveira[b]
[a] *Departamento de Física e Química, Universidade Estadual Paulista,*
Av. Ariberto Pereira da Cunha 333, Guaratinguetá, 12500-000, São Paulo, SP, Brazil,
e-mail: everton@feg.unesp.br
[b] *Departamento de Física, ICE, Universidade Federal de Juiz de Fora,*
Juiz de Fora, $36036-330$, *Minas Gerais, MG, Brazil,*
e-mail: jorge@fisica.ufjf.br and wilson@fisica.ufjf.br

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The $SU(2)$ Skyrme model,expanding in the collective coordinates variables, gives rise to second-class constraints. Recently this system was embedded in a more general Abelian gauge theory using the BFFT Hamiltonian method. In this work we quantize this gauge theory computing the Noether current anomaly using for this two different methods: an operatorial Dirac first class formalism and the non-local BV quantization coupled with the Fujikawa regularization procedure.

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I. INTRODUCTION

The field-antifield formalism, created by I. Batalin and G. Vilkovisky (BV method) [1] has been used successfully to quantize the most difficult gauge theories such as supergravity theories and topological field theories in the Lagrangian framework [2–4]. The BV method comprises the Faddeev-Popov quantization [5] and has the BRST symmetry as fundamental principle [6]. The method has introduced the definition of the antifield which are the sources of the BRST transformation, i.e., for each field we have an antifield canonically conjugated in terms of the antibracket operation. With the fields, the antifields and the BRST transformation we can construct the classical BV action. A mathematical ingredient, called the antibracket, helps us to construct the fundamental equation of the formalism at the classical level, the so-called master equation.

At the quantum level we can define another mathematical operator, the Δ operator, which is a second order differential operator. With the classical BV action and presenting local counterterms, we can construct the quantum BV action and analogously to the classical case, the quantum master equation.

The quantization is performed via the usual functional integration through the definition of the generating functional with the help of the well known Legendre transformation with the respect to the sources J_A. When it is not possible to find a solution to the quantum master equation we can say that the theory has an anomaly. The presence of a $\delta(0)$ term in the Δ operation demand a method to treat this divergence conveniently. There are various methods to regularize the theory such as Pauli-Villars [7], BPHZ [8,9] and dimensional regularization [10]. Recently, the non-local regularization [11,12] coupled with the field-antifield formalism [13–15] has been developed. The success of the last one is based on its power to compute the anomaly of higher-loops. The BRST superspace formulation brings another construction of the main ingredients of BV formalism [16].

The Skyrme model was first proposed by T. H. R. Skyrme [17] in the sixties to incorporate baryons in the non-linear sigma model description of the low-energy interactions of pions [18]. A quantum theory can be constructed through the definition of the physical states which are annihilated by operators of the first-class constraints, and then we can obtain the physical values after taking the mean value of the canonical operators.

Some efforts have been performed to quantize the Skyrme model. Two of us [19] have achieved this by applying the non-abelian BFFT [19,20] formalism, and thus, employ the Dirac method of first-class constraints to quantize the system [21]. The analysis of the physical spectrum as well as the study of a hidden symmetry over the ordering ambiguity problem in the Skyrme model is discussed in [22]. A formulation of the model as an embedded gauge theory with the constraint deformed away from the spherical geometry is proposed in [23].

Generally the anomaly is related to the fermionic aspect of the Skyrme-soliton physics or with the model coupled to fermions. The first-class bosonic model has the auxiliary fields firstly introduced by L. D. Faddeev [24] to convert a second-class system in a first-class one. In [25], the idea of adding extra degrees of freedom has been implemented in the BV scheme. Furthermore in [26], it was shown that the cohomology of the classical theory has not been changed by the introduction of these new degrees of freedon. As a consequence, the anomaly has not disappeared, but is shifted to these extra symmetries. Hence we can say that the anomaly is in fact hidden in this way.

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The purpose of this paper is to compute, in a precise way, the value of the anomaly of the bosonic model using an operatorial Dirac first-class formalism and the non-local field-antifield formalism coupled with the Fujikawa procedure of regularization [27]. Finally we want to introduce some ideas about the geometry involved in the Skyrme model's anomaly. The paper is organized as follow: in section 2, the first class Skyrme action was derived and the operatorial calculation of the anomaly is developed; we have made a brief review of the non-local BV method in section 3; in section 4, we compute the one-loop anomaly. The final considerations have been made in section 5.

II. THE SKYRME MODEL AND THE OPERATORIAL METHOD

The classical static Lagrangian of the Skyrme model [17] is given by

$$L = \int d^3r \{ -\frac{F_\pi^2}{16} Tr\left(\partial_i U \partial_i U^+\right) + \frac{1}{32e^2} Tr\left[U^+\partial_i U, U^+\partial_j U\right]^2 \}, \tag{1}$$

where F_π is the pion decay constant, e is a dimensionless parameter and U is an SU(2) matrix. Performing the collective semi-classical expansion [28], substituting U(r) by $U(r,t) = A(t)U(r)A^+(t)$ in (1), where A is an SU(2) matrix, we obtain

$$L = -M + \lambda Tr[\partial_0 A \partial_0 A^{-1}], \tag{2}$$

where M is the soliton mass, which in the hedgehog representation for U, $U = \exp(i\tau \cdot \hat{r} F(r))$, is given by

$$M = 4\pi \frac{F_\pi}{e} \int_0^\infty dx\, x^2 \left\{ \frac{1}{8}\left(F'^2 + \frac{2\sin^2 F}{x^2}\right) + + \frac{1}{2}\left[\frac{\sin^2 F}{x^2}\left(\frac{\sin^2 F}{x^2} + 2F'^2\right)\right] \right\}, \tag{3}$$

where x is a dimensionless variable defined by $x = eF_\pi r$ and λ is called the inertia moment written as $\lambda = \frac{2}{3}\pi(e^3 F_\pi)^{-1}\Lambda$ with

$$\Lambda = \int_0^\infty dx\, x^2 \sin^2 F \left[1 + 4(F'^2 + \frac{\sin^2 F}{x^2})\right]. \tag{4}$$

The SU(2) matrix A can be written as $A = a^0 + ia \cdot \tau$ with the constraint

$$T_1 = a^i a^i - 1 \approx 0, \quad i = 0,1,2,3. \tag{5}$$

The Lagrangian(2) can be written as a function of the a^i as

$$L = -M + 2\lambda \dot{a}^i \dot{a}^i. \tag{6}$$

Batalin, Fradkin, Fradikina and Tyutin [20] developed an elegant formalism of transforming systems with second class constraints in first class ones, i.e., in gauge theories. This is achieved with the aid of auxiliary fields that extend the phase space in a convenient way to transform the second class into first class constraints. This formalism and its extension non-Abelian [32] were used for transforming the $SU(2)$ Skyrme model in an Abelian and non-Abelian gauge theory [19], respectively. Since a Abelian gauge theory, the corresponding Lagrangian is given by

$$\tilde{L} = -M + 2\lambda \frac{\dot{a}^i \dot{a}^i}{a^i a^i} - 2\lambda \frac{\dot{\phi}\dot{\phi}}{(a^i a^i)^2}, \tag{7}$$

where ϕ^i are the auxiliary fields of BFFT formalism.

The new set of first class constraints are given by

$$\tilde{T}_1 = T_1 + 2\phi, \tag{8}$$
$$\tilde{T}_2 = T_2 - a^i a^i \pi_\phi. \tag{9}$$

The action is written as

$$S = \int dt \left[-M + 2\lambda \frac{\dot{a}^i \dot{a}^i}{a^i a^i} - 2\lambda \frac{\dot{\phi}\dot{\phi}}{\left(a^i a^i\right)^2} \right], \tag{10}$$

which is invariant for the following gauge transformations

$$\delta a^j = \tau a^j, \tag{11}$$
$$\delta\phi = 2\tau\phi, \tag{12}$$

where τ is a constant parameter or a function dependent on position. The Noether current is obtained by using the formula [31]

$$j_0 = \frac{\delta L}{\delta \dot{a}^i}\delta a^i + \frac{\delta L}{\delta \dot{\phi}}\delta\phi, \tag{13}$$

which result in

$$j_0 = a^i \pi^i + 2\phi\pi_\phi. \tag{14}$$

In the first class Dirac quantization constraints method [21], the physical wave functions are obtained by imposing the condition

$$\tilde{T}_\alpha |\psi\rangle_{phys} = 0, \quad \alpha = 1,2, \tag{15}$$

being the operators $\tilde{T}^1$ and $\tilde{T}^2$ given by

$$\tilde{T}^1 = a^i a^i - 1 + 2\phi, \tag{16}$$
$$\tilde{T}^2 = a^i \pi^i - a^i a^i \pi_\phi. \tag{17}$$

Then, the physical states that satisfy (15) are

$$|\psi\rangle_{phys} = = \delta(a^i\pi^i - a^i a^i \pi\phi)\,\delta(a^i a^i - 1 + 2\phi)\,|polynomial\rangle, \tag{18}$$

where the ket *polynomial* is defined as $|polynomial\rangle = \frac{1}{N(l)}(a^1 + ia^2)^l$, and $N(l)$ is a normalization factor. The polynomial kets are the typical skyrmion collective coordinates eigenstates [28].

Taking the scalar product, ${}_{phys}\langle\psi|j_0|\psi\rangle_{phys}$, that is the mean value of the Noether current, formula (14), we have

$$
\begin{aligned}
&{}_{phys}\langle\psi|j_0|\psi\rangle_{phys} = \\
&= \langle polynomial| \int d\phi\, d\pi_\phi \cdot \\
&\cdot \delta(a^ia^i - 1 + 2\phi)\delta(a^i\pi^i - a^ia^i\pi_\phi) \cdot \\
&\cdot j_0\delta(a^ia^i - 1 + 2\phi)\delta(a^i\pi^i - a^ia^i\pi_\phi)|polynomial\rangle. \quad (19)
\end{aligned}
$$

Note that due to $\delta(a^ia^i - 1 + 2\phi)$ and $\delta(a^i\pi^i - a^ia^i\pi_\phi)$ in (19), the scalar product can be simplified. Then, integrating over ϕ and π^ϕ we obtain

$$
\begin{aligned}
&{}_{phys}\langle\psi|\ j_0\ |\psi\rangle_{phys} = \\
&\quad = \langle polynomial|\frac{a^i\pi^i}{a^ia^i}|polynomial\rangle. \qquad (20)
\end{aligned}
$$

The operator π^j describes a free momentum particle and its representation on the collective coordinates space a_i is given by

$$\pi_j = -i\frac{\partial}{\partial a_j}. \qquad (21)$$

The current operator inside the ket (20) must be symmetrized. For this we use the Weyl ordering prescription [29]. This rule expresses that the new current operator must be constructed by counting all possible randomly order of the a^i, π^i and $1/(a^ia^i)$. Then, we can write the symmetric current as

$$\left[\frac{a^i\pi^i}{a^ia^i}\right]_{sym} = \frac{1}{a^ia^i}a^j\pi^j - \frac{i}{a^ia^i}, \qquad (22)$$

where the ordering of the operator $a^j\pi^j/(a^ia^i)$ means that $a^j\pi^j$ acts firstly in the physical kets. The polynomial states are eigenstates of the operator $a^i\pi^i$, i.e., $a^i\pi^i|polynomial\rangle = l|polynomial\rangle$. Thus, the mean values of j_0 is given by[1]

$$
\begin{aligned}
&{}_{phys}\langle\psi|j_0|\psi\rangle_{phys} = \\
&= \frac{-i(l+1)}{V^2}\langle polynomial|\frac{1}{a^ia^i}|polynomial\rangle \\
&= \frac{-i(l+1)}{V^2}\frac{1}{a^ia^i}. \qquad (23)
\end{aligned}
$$

In the above expressions, a^ia^i is the square of the three-sphere collective coordinates radius. Then, the polynomial eigenstates do not depend of the term $1/(a^ia^i)$.

Moreover, the term $1/(a^ia^i)$ in the definition of the mean value of j_0, eq. (23), is an "curvature scalar " of the hypersurface defined by the second class constraint T_1, eq. (5) and, at first, in the first class system, it is not conserved in time. This fact can indicate a possible anomaly in the Noether current j_0.

The current anomaly can be calculated by an scalar product given by

$$
\begin{aligned}
\mathcal{A} &= \int dt\, {}_{phys}\langle\psi|\partial_0 j_0|\psi\rangle_{phys} \\
&= \frac{1}{V^2}\int dt\langle polynomial|\partial_0\left[\frac{a^i\pi^i}{a^ia^i}\right]_{sym}|polynomial\rangle \\
&= \frac{-i(l+1)}{V^2}\int dt\langle polynomial|\partial_0(1/a^ia^i)|polynomial\rangle.
\end{aligned}
$$

Using the definition of the momentum obtained through the Lagrangian written in the action(10), $\dot{a}^i = \frac{a^ja^j}{4\lambda}\pi^i$, the anomaly $\mathcal{A}$ can be written as

$$\mathcal{A} = \frac{i\,(l\,+\,1\,)}{2\,\lambda\,V^2}\int dt\langle polynomial|\left[\frac{a^i\pi^i}{a^ia^i}\right]_{sym}|polynomial\rangle.$$

Symmetrized the operator $a^i\pi^i/(a^ia^i)$ written in the expression above, using for this again the Weyl ordering prescription, and considering that the ket *polynomial* is an eigenstate of the operator $a^i\partial^i$, we can finally obtain the expression for the anomaly $\mathcal{A}$, given by

$$\mathcal{A} = \frac{(l\,+\,1\,)^2}{2\,\lambda\,V^2}\int dt\frac{1}{a^ia^i}\ , \qquad (24)$$

and from this expression we can see that the constant terms have to be computed. Next we will do this using the BV nonlocal method finding the whole expression of this anomaly.

III. THE FIELD-ANTIFIELD FORMALISM WITH NON-LOCAL REGULARIZATION

Let us construct the complete set of fields, Φ^A, including in this set the classical fields, the ghosts for all gauge symmetries and the auxiliary fields. We can also define the antifields Φ^*_A, which are the canonical conjugated variables with respect to the antibracket structure,

$$(X,Y) = \frac{\delta_r X}{\delta\phi}\frac{\delta_l Y}{\delta\phi^*} - (X \longleftrightarrow Y)\ , \qquad (25)$$

where the indices r and l denote, as usual, right and left functional derivatives respectively.

One can then construct an extended action of ghost number equal to zero, the so-called BV action, also called classical proper solution,

$$
\begin{aligned}
S(\Phi,\Phi^*) &= S_{cl}(\Phi) + \Phi^*_A R^A(\Phi) + \frac{1}{2}\Phi^*_A\Phi^*_B R^{BA}(\Phi) \\
&\quad + \ldots + \frac{1}{n!}\Phi^*_{A_1}\ldots\Phi^*_{A_n}R^{A_n\ldots A_1} + \ldots\ , \qquad (26)
\end{aligned}
$$

[1] The regularization of delta function squared like $\delta(a^ia^i - 1+2\phi)^2$ and $\delta(a^i\pi^i - a^ia^i\pi_\phi)^2$ is performed by using the delta relation, $(2\pi)^2\delta(0) = \lim_{k\to 0}\int d^2x\, e^{ik\cdot x} = \int d^2x = V$.

where $R^A(\Phi)$ are the BRST generators. This action has to satisfy the classical master equation, $(S,S)=0$. At the quantum level the action can be defined by $W = S + \sum_{p=1}^{\infty} \hbar^p M_p$, where the M_p are the corrections (the Wess-Zumino terms) to the quantum action. The quantization of the theory is obtained with the generating functional of the Green functions:

$$Z(J,\Phi^*) = \int \mathcal{D}\Phi \, exp \, \frac{i}{\hbar} \left[W(\Phi,\Phi^*) + J^A \Phi^*_A \right] . \qquad (27)$$

But the definition of a path integral properly lacks on a regularization framework.

For a theory to be free of anomalies, the quantum action W has to be a solution of the QME, $(W,W) = 2\,i\,\hbar\,\Delta\,W$, where $\Delta \equiv (-1)^{A+1} \frac{\partial_r}{\partial \Phi^A} \frac{\partial_r}{\partial \Phi^*_A}$. In the QME one can see that when it is not possible to find a solution, we have an anomaly that can be defined by

$$\mathcal{A} \equiv \left[\Delta W + \frac{i}{2\hbar}(W,W) \right] (\Phi,\Phi^*) \quad . \qquad (28)$$

To accomplish the regularization we will choose a method developed recently [11,12] and that has been adapted to the BV formalism [13,14] [2]. Let us define an action $S(\Phi)$ as being, $S(\Phi) = F(\Phi) + I(\Phi)$, where $F(\Phi)$ is the kinetic part and $I(\Phi)$ is the interacting part, which is an analytic function in Φ^A around $\Phi^A = 0$. Then one can write conveniently that $F(\Phi) = \frac{1}{2} \Phi^A \mathcal{F}_{AB} \Phi^B$, where $\mathcal{F}_{AB}$ is called the kinetic operator. To perform the NLR we have now to introduce a cut-off or regulating parameter Λ^2. An arbitrary and invertible matrix T_{AB} has to be introduced too. The combination of $\mathcal{F}_{AB}$ with $(T^{-1})^{AB}$ defines a second order derivative regulator, $\mathcal{R}^A_B = (T^{-1})^{AC} \mathcal{F}_{CB}$.

We can construct two important operators with these objects. The first is the smearing operator $\epsilon^A_B = exp\left(\frac{\mathcal{R}^A{}_B}{2\Lambda^2} \right)$, and the second is the shadow kinetic operator

$$\mathcal{O}^{-1}_{AB} = T_{AC} (\tilde{\mathcal{O}}^{-1})^C{}_B = \left(\frac{\mathcal{F}}{\epsilon^2 - 1} \right)_{AB} \quad , \qquad (29)$$

with $(\tilde{\mathcal{O}})^A_B$ defined as

$$\tilde{\mathcal{O}}^A{}_B = \left(\frac{\epsilon^2 - 1}{\mathcal{R}} \right)^A{}_B = \int_0^1 \frac{dt}{\Lambda^2} \, exp \left(t \, \frac{\mathcal{R}^A_B}{\Lambda^2} \right) \quad . \qquad (30)$$

In order to expand our original configuration space for each field Φ^A, an auxiliary field Ψ^A can be constructed,

$$\tilde{\mathcal{S}}(\Phi,\Psi) \, = \, F\,(\hat{\Phi}) \, - \, A\,(\Psi) \, + \, I\,(\Phi + \Psi) \quad . \qquad (31)$$

[2] For convenience, in this quite brief review, we are using the same notation as the reference [13].

The second term of this auxiliary action is called the auxiliary kinetic term, $A(\Psi) = \frac{1}{2} \Psi^A (\mathcal{O}^{-1})_{AB} \Psi^B$. The fields $\hat{\Phi}^A$, the smeared fields, which make part of the auxiliary action are defined by $\hat{\Phi}^A \equiv (\epsilon^{-1})^A{}_B \Phi^B$.

In the non-local BV formalism the configuration space has to be enlarged introducing the antifields $\{\Psi^A, \Psi^*_A\}$. Note that the shadow fields have antifields too. Then, an auxiliary proper solution, $\tilde{S}(\Phi,\Phi^*;\Psi,\Psi^*)$, incorporates the auxiliary action (31) (corresponding to the gauge-fixed action $S(\Phi)$), its gauge symmetry and the unknown associated higher order structure functions. The auxiliary BRST transformations are modified by the presence of the term $\Phi^*_A \, R^A(\Phi)$ in the original proper solution. The shadow fields have to be substituted by the solutions of their classical equations of motion. At the same time, their antifields will be equal to zero. In this way we can write $S_\Lambda(\Phi,\Phi^*) = \tilde{S}(\Phi,\Phi^*;\Psi,\Psi^* = 0)$, and the classical equations of motion are $\delta_r \, \tilde{S}(\Phi,\Phi^*;\Psi,\Psi^*)/\delta\Psi^A = 0$ with solutions $\bar{\Psi} \equiv \bar{\Psi}(\Phi,\Phi^*)$.

The complete interaction term, $\mathcal{I}(\Phi,\Phi^*)$, of the original proper solution can be written as

$$\mathcal{I}(\Phi,\Phi^*) \equiv I(\Phi) \, + \, \Phi^*_A \, R^A(\Phi) \, + \, \Phi^*_A \, \Phi^*_B \, R^{BA}(\Phi) \, + \ldots \qquad (32)$$

and the quantum action W can be expressed by $W = F + \mathcal{I} + \sum_{p=1}^{\infty} \hbar M_p \equiv F + \mathcal{Y}$, where $\mathcal{Y}$ is the generalized quantum interaction part. It can be shown that the expression of the anomaly is the value of the finite part in the limit $\Lambda^2 \longrightarrow \infty$ of

$$\mathcal{A} = \left[(\,\Delta\,W\,)_R \, + \, \frac{i}{2\,\hbar}\,(W,W) \right] \, (\Phi,\Phi^*) \qquad (33)$$

and the regularized value of ΔW is defined as $(\Delta W)_R \equiv \lim_{\Lambda^2 \to \infty} [\Omega_0]$ where Ω_0 and $(\delta_\Lambda)^A{}_B$ are defined by

$$\begin{aligned} \Omega_0 &= \left[S^A{}_B \, (\delta_\Lambda)^B{}_C \, \left(\epsilon^2\right)^C{}_A \right] , \\ (\delta_\Lambda)^A{}_B &= \left(\delta^A{}_B - \mathcal{O}^{AC} \, \mathcal{I}_{CB} \right)^{-1} \\ &= \delta^A{}_B + \sum_{n=1} \, \left(\mathcal{O}^{AC} \, \mathcal{I}_{CB} \right)^n \quad , \end{aligned} \qquad (34)$$

with $S^A{}_B = \delta_r \, \delta_l \, S/\delta\,\Phi^B \,\delta\,\Phi^*_A$, $\mathcal{I}_{AB} = \delta_r \, \delta_l \, \mathcal{I}/\delta\,\Phi^A \,\delta\,\Phi^B$. Applying the limit $\Lambda^2 \longrightarrow \infty$ in $(\Delta W)_R$, it can be shown that $(\Delta S)_R \equiv \lim_{\Lambda^2 \to \infty} [\Omega_0]_0$, and finally that $\mathcal{A}_0 \equiv (\Delta\,S)_R = \lim_{\Lambda^2 \to \infty} [\Omega_0]_0$. All the higher orders terms of the anomaly can be obtained from equation (33), but this will not be analyzed in this paper. It can be seen in [14].

IV. THE NON-LOCAL FIELD-ANTIFIELD QUANTIZATION OF THE SKYRME MODEL

The first-class action for the massless Skyrme model is, using (7),

$$\mathcal{S} \;=\; \int dt \, \left[\frac{2\,\lambda\,\dot{a}^i\,\dot{a}^i}{a^i\,a^i} \;-\; \frac{2\,\lambda\,\dot{\phi}^i\,\dot{\phi}^i}{(\,a^i\,a^i\,)^2} \right] \quad , \tag{35}$$

but we already know that the first-class tell us that $2\phi = 1 - a^i a^i$ so that $\dot{\phi} = -a^i \dot{a}^i$. Substituting this in (35) we have now that

$$\mathcal{S} \;=\; \int dt \, \left[\frac{2\,\lambda\,\dot{a}^i\,\dot{a}^i}{a^i\,a^i} \;-\; \frac{2\,\lambda\,(a^i\dot{a}^i)^2}{(\,a^i\,a^i\,)^2} \right] \quad , \tag{36}$$

This action, as we can easily see, has a problem of non-locality, which can be solved expanding the terms,

$$\begin{aligned}
\mathcal{S} \;&=\; \int dt \, \left\{ \frac{2\,\lambda\,\dot{a}^i\,\dot{a}^i}{[1-(1-a^i\,a^i)]} \;-\; \frac{2\,\lambda\,(a^i\dot{a}^i)^2}{[1-(1-\,a^i\,a^i)]^2} \right\} \\
&= 2\lambda \dot{a}^i\,\dot{a}^i \sum_{n=0}^{\infty} (1-a^i\,a^i)^n \\
&-\; 2\lambda(\,a^i\,\dot{a}^i\,)^2 \sum_{n=0}^{\infty} (n+1)(1-a^i\,a^i)^n
\end{aligned} \tag{37}$$

This action is invariant for the BRST transformations given by [30]

$$\delta a^i \;=\; c\,a^i\;, \qquad \delta c \;=\; 0 \quad . \tag{38}$$

Now we can construct the BV action,

$$\begin{aligned}
\mathcal{S}_{BV} \;=\; \int dt \, \Bigg\{ & 2\lambda \dot{a}^i\,\dot{a}^i \sum_{n=0}^{\infty} (1-a^i\,a^i)^n \\
&-\; 2\,\lambda\,(\,a^i\,\dot{a}^i\,)^2 \sum_{n=0}^{\infty} (n+1)(1-a^i\,a^i)^n \;+\; a_i^* c a^i \Bigg\}
\end{aligned} \tag{39}$$

The kinetic part of the action (35) after an integration by parts is,

$$F = \int d\,t\,[\,2\,\lambda\,\dot{a}^i\,\dot{a}^i\,] = \int dt[\,-2\,\lambda\,a^i\,\partial_0^2\,a^i\,] \quad . \tag{40}$$

The kinetic term has the form

$$F = \frac{1}{2} a^i (-4\lambda\partial_0^2)\,a^i \quad \Longrightarrow \quad \mathcal{F}_{AB} \;=\; -4\,\lambda\,\partial_0^2\,\delta_{AB} \tag{41}$$

The regulator, a second order differential operator, can be chosen as

$$\mathcal{R}^A_B = \partial_0^2 \quad , \quad \Longrightarrow \quad T \;=\; -4\lambda \quad . \tag{42}$$

where T is an arbitrary non-singular matrix. The smearing operator has the form, $\epsilon^A{}_B \;=\; exp\left(\frac{\partial_0^2}{2\,\Lambda^2}\right)\,\delta^A{}_B$. In the NLR scheme the shadow kinetic operator is

$$\mathcal{O}^{-1}_{AB} = \left(\frac{\mathcal{F}}{\epsilon^2 - 1} \right)_{AB} = \left(\frac{-4\lambda\partial_0^2}{\epsilon^2 - 1} \right)_{AB} \tag{43}$$

where

$$\mathcal{O}^{AB} = -\,\frac{\epsilon^2 - 1}{4\,\lambda\,\partial_0^2} \;=\; -\,\int_0^1 \frac{d\tau}{\Lambda^2}\,exp\left(\tau \frac{4\,\partial_0^2}{\Lambda^2} \right) \tag{44}$$

Using the definitions of $S^A{}_B$ and $\mathcal{I}_{AB}$ we can show that,

$$S^a{}_a \;=\; c \tag{45}$$

$$\begin{aligned}
\mathcal{I}_{aa} = & -4\,\lambda\,\partial_0^2 \;+\; \frac{4\,\lambda\,\partial_0^2}{a^i\,a^i} \;-\; \lambda\frac{(16 a^i \dot{a}^i \partial_0 \;+\; \dot{a}^i \dot{a}^i)}{(a^i\,a^i)^2} \\
& -\lambda \frac{(\dot{a}^i\dot{a}^i \;+\; 12\,a^i\dot{a}^i\partial_0 \;+\; (a^i\partial_0)^2)}{(a^i\,a^i)^2} \;+\; 8\lambda\frac{\dot{a}^i\dot{a}^i a^j\,a^j}{(a^i\,a^i)^3} \\
& -\; 8\lambda\frac{(\dot{a}^i\dot{a}^i)^2}{(a^i\,a^i)^4}
\end{aligned} \tag{46}$$

Finally the anomaly can be computed as we know

$$\begin{aligned}
\mathcal{A} \;&=\; (\Delta S)_R \\
&=\; \lim_{\Lambda^2 \to \infty} \{ Tr[\epsilon^2 S^A{}_B] \;+\; Tr[\epsilon^2 S^A{}_D \mathcal{O}^{DC} \mathcal{I}_{CB}] \}
\end{aligned} \tag{47}$$

Computing each term we have, for the first term in $\mathcal{I}_{aa}$, writing only the main steps,

$$\begin{aligned}
& \lim_{\Lambda^2 \to \infty} \left[-4\,\epsilon^2\,\lambda\,c \int dt\,\mathcal{O}\,\partial_0^2 \right] \\
&= \lim_{\Lambda^2 \to \infty} \left[-4\,\epsilon^2\,\lambda\,c \int dt \int \frac{dk}{2\pi} e^{-ikt}\,\mathcal{O}\partial_0^2 exp\left(\frac{\partial_0^2}{\Lambda^2}\right) e^{ikt} \right] \\
&= \lim_{\Lambda^2 \to \infty} \Bigg[-4\,\epsilon^2\,\lambda\,c \int \frac{dt}{\Lambda} \cdot \\
& \cdot \int_0^1 \left(-\frac{d\tau}{\Lambda^2} \right) exp\left(\tau \frac{4\,\partial_0^2}{\Lambda^2} \right) \cdot \int \frac{dk}{2\pi} (-\,k^2) exp\left(\frac{-k^2}{\Lambda^2} \right) \Bigg] \\
&= \lim_{\Lambda^2 \to \infty} \left[-4\,\epsilon^2\,\lambda\,c \int \frac{dt}{\Lambda} \int d\tau \left(1 + \frac{4\,\partial_0^2}{\Lambda^2} \right) \left(-\frac{\sqrt{\pi}}{2} \right) \right] \\
&= \lim_{\Lambda^2 \to \infty} \left[-4\,\epsilon^2\,\lambda\, \left(-\frac{\sqrt{\pi}}{2} \right) \int dt\,c \right]
\end{aligned} \tag{48}$$

where we have made two convenient reparametrizations $(t, k) \to (\lambda t, \lambda k)$ to solve the integrals [33].

Analogously for another term in $\mathcal{I}_{aa}$

$$\begin{aligned}
& \lim_{\Lambda^2 \to \infty} \quad \left\{ 3\,\epsilon^2\,\lambda\,c \int dt\,\mathcal{O}\,\frac{\partial_0^2}{a^i\,a^i} \right\} \;= \\
& \qquad = \lim_{\Lambda^2 \to \infty} \left\{ \frac{3}{2} \sqrt{\pi}\,\epsilon^2\,\lambda \int dt \frac{c}{a^i\,a^i} \right\} \quad .
\end{aligned} \tag{49}$$

Doing the same procedure for all the other terms one can conclude that they are identically zero.

As we know, terms that depends only on ghots does not have any physical meaning in the final result of the anomaly. Computing only the physical terms, the one-loop anomaly for the $SU(2)$ Skyrme model is the Wess-Zumino consistent expression [34],

$$\mathcal{A} = \frac{3}{2} \sqrt{\pi}\,\lambda \int dt\,\frac{c}{a^i\,a^i} \quad . \tag{50}$$

It is a new result, corroborating the general expression founded before in eq. (24) showing an anomaly in the conservation of the Noether current j_0. But at the level of BV formalism, as well known, it represents an impossibility to the solution of the QME.

V. CONCLUSIONS

In this work we have considered the $SU(2)$ Skyrme model as a more general Abelian gauge theory. Firstly we have quantized this gauge theory using an operatorial Dirac first-class formalism and computing the anomaly of the Noether current as a manifestation of the Gaussian curvature of the hypersurfaces of constraints. But this method yields an expression dependent on unknown geometrical constant terms. It is the nonlocal field-antifield formalism which discloses the whole anomaly expression.

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