Pairing effects in low density domain of nuclear matter

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Using equations, governing np pairing correlations in $S = 1, T = 0$ pairing channel (PRC 63 (2001) 021304(R)), it is shown that at low densities equations for the energy gap in the spectrum of quasiparticles and chemical potentials of protons and neutrons allow solutions with negative chemical potential. This corresponds to appearance of Bose–Einstein condensate (BEC) of deuterons in low density region of nuclear matter.

The transition from BCS superconductivity to Bose–Einstein condensation occurs in a Fermi system, if either density is decreased or the attractive interaction between fermions is increased sufficiently. Recently it was realized that this phase transition takes place in symmetric nuclear matter, when np Cooper pairs at higher densities go over to BEC of deuterons at lower densities [1, 2]. During the phase transition the chemical potential changes its sign at certain critical density (Mott transition), approaching half of the deuteron binding energy at ultra low densities. Here for studying corresponding phase transition in asymmetric nuclear matter we shall use equations, obtained in Ref. [3] for description of np pairing correlations in $S = 1, T = 0$ pairing channel:

$$\Delta(\mathbf{k}) = -\frac{1}{V}\sum_{\mathbf{k}'} V(\mathbf{k},\mathbf{k}')\frac{\Delta(\mathbf{k}')}{2E_{k'}}(1 - f(E^+_{k'}) - f(E^-_{k'})), \quad (1)$$

$$\varrho = \frac{2}{V}\sum_{\mathbf{k}}\Big(1 - \frac{\varepsilon_k}{E_k}[1 - f(E^+_k) - f(E^-_k)]\Big) \equiv \frac{2}{V}\sum_{\mathbf{k}} n_k, \quad (2)$$

$$\alpha\varrho = \frac{2}{V}\sum_{\mathbf{k}}\Big(f(E^+_k) - f(E^-_k)\Big),\ E^{\pm}_k = E_k \pm \mu_{03} = \sqrt{\varepsilon^2_k + \Delta^2(\mathbf{k})} \pm \mu_{03}, \quad (3)$$

where $f(E)$ is Fermi distribution, ε_k is the single–particle spectrum, μ_{00}, μ_{03} being half of a sum and half of a difference of proton and neutron chemical potentials. Eq. (1) is equation for the energy gap Δ and Eqs. (2), (3) are equations for the total density

*A.I. is grateful for support of Topical Program of APCTP during his stay at Seoul.
†J.Y. was partially supported by Korea Research Foundation Grant (KRF-2001-041-D00052).

$\varrho = \varrho_p + \varrho_n$ and neutron excess $\delta\varrho = \varrho_n - \varrho_p \equiv \alpha\varrho$ (α being the asymmetry parameter). Then, introducing the anomalous density

$$\psi(\mathbf{k}) =< a^+_{p,k} a^+_{n,-k} >= \frac{\Delta(\mathbf{k})}{2E_k}\Big(1 - f(E^+_k) - f(E^-_k)\Big)$$

and using Eq. (2), one can represent Eq. (1) for the energy gap in the form

$$\frac{k^2}{m}\psi(\mathbf{k}) + (1 - n_k)\sum_{\mathbf{k}'} V(\mathbf{k}, \mathbf{k}')\psi(\mathbf{k}') = 2\mu_{00}\psi(\mathbf{k}). \tag{4}$$

In the limit of vanishing density, $n_k \to 0$, Eq. (4) goes over into the Schrödinger equation for the deuteron bound state [2, 4]. Corresponding energy eigenvalue is equal to $2\mu_{00}$.

Further for numerical calculations we shall use the effective zero range force, developed in Ref. [5] to reproduce the pairing gap in $S = 1, T = 0$ pairing channel with Paris NN potential:

$$V(\mathbf{r}_1, \mathbf{r}_2) = v_0\Big\{1 - \eta\Big(\frac{\varrho(\frac{\mathbf{r}_1+\mathbf{r}_2}{2})}{\varrho_0}\Big)^\gamma\Big\}\delta(\mathbf{r}_1 - \mathbf{r}_2), \tag{5}$$

where ϱ_0 is the nuclear saturation density ($\varrho_0 = 0.16\,\mathrm{fm}^{-3}$), v_0, η, β are some adjustable parameters. Besides, in the gap equation (1), Eq. (5) must be supplemented with a cut–off parameter ε_c. We utilize the following set of parameters: $\eta = 0$, $v_0 = -530\,\mathrm{MeV}\cdot\mathrm{fm}^3$, $m = m_G$, $\varepsilon_c = 60\,\mathrm{MeV}$, where m_G is the effective mass, corresponding to the Gogny force D1S [

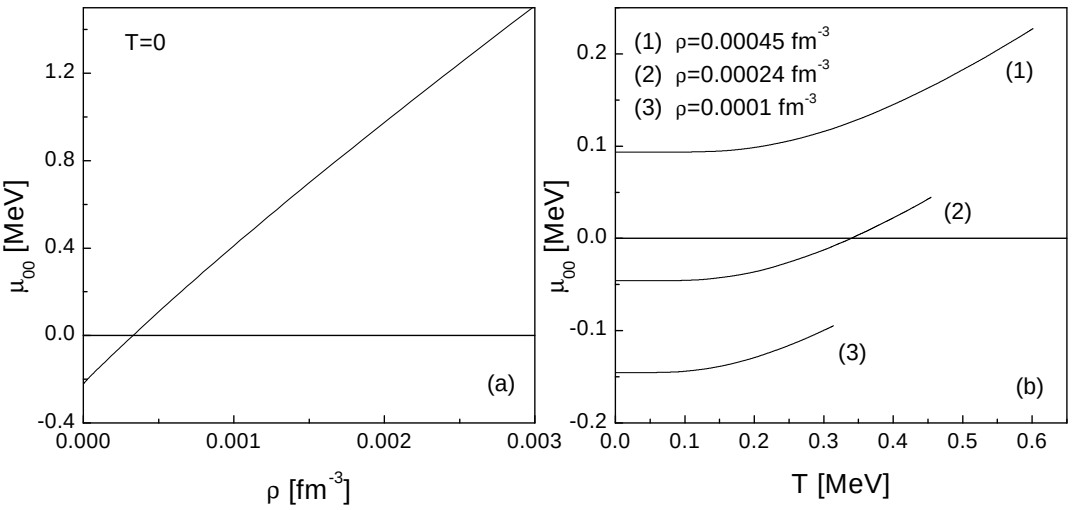

Figure 1. Chemical potential μ_{00} as a function of: (a) density at zero temperature, (b) temperature at fixed densities (low density region).

6]. This set was developed to produce the bound state between two extreme values: at zero energy and at the deuteron binding energy [5].

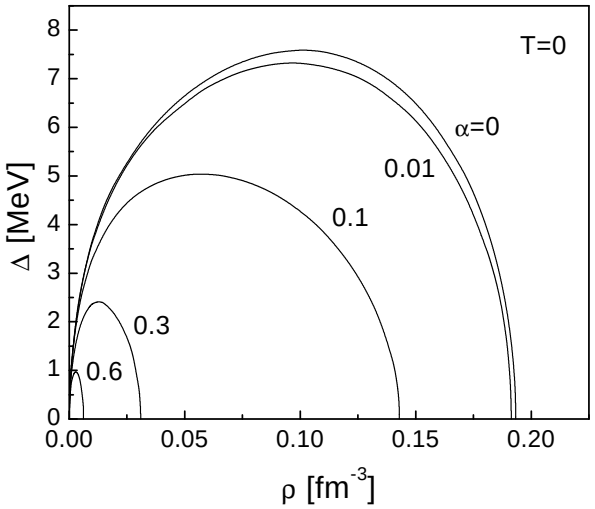

Figure 2. Energy gap as a function of density at zero temperature and different asymmetries.

First we consider the case of symmetric nuclear matter ($\alpha = 0$). In Fig. 1(a) it is shown the zero temperature behavior of the chemical potential μ_{00}. It is seen that the chemical potential at some density ϱ_b ($\varrho_b \approx 3{\cdot}10^{-4}\,\mathrm{fm}^{-3}$) changes its sign, that, according to Eq. (4), corresponds to appearance of deuteron–like bound states in nuclear matter. When density tends to zero, the chemical potential approaches its asymptotic value $\mu_{00} = -\varepsilon_b/2$, ε_b being the binding energy of a bound state. Therefore, we can conclude that under lowering density np superfluidity smoothly evolves into the BEC of np bound states (deuterons).

In Fig. 1(b) we present the results of numerical determination of the temperature dependence of the chemical potential μ_{00} for the fixed values of density. If density is low enough then under decrease of temperature the chemical potential becomes negative (the curve 2), that corresponds to formation of bound states. At very low densities np condensate exists only in the form of BEC of deuteron–like bound states (the curve 3). If densities is, however, high enough, np Cooper pairs survive even at zero temperature (the curve 1).

Now we consider asymmetric nuclear matter. The results of numerical calculations for the energy gap as a function of density for different asymmetries at zero temperature are shown in Fig. 2. As one can see, with increasing asymmetry the value of the energy gap decreases and the density interval, where np pairs exist, shrinks in the direction of zero density. In reality solutions exist for any $\alpha < 1$ (the phase curves for larger values of α are not shown in Fig. 2) and corresponding density interval contracts more and more to the point $\varrho = 0$, when asymmetry increases. To understand why the isospin asymmetry loses its efficiency in destroying np pairing correlations in low density region, let us note that at zero temperature the contribution to the integral in the gap equation gives the interval $[0, \varepsilon_c]$, excluding the domain $[\mu_{00} - \Delta\varepsilon, \mu_{00} + \Delta\varepsilon]$, where $\Delta\varepsilon = \sqrt{\mu_{03}^2 - \Delta^2}$. In the

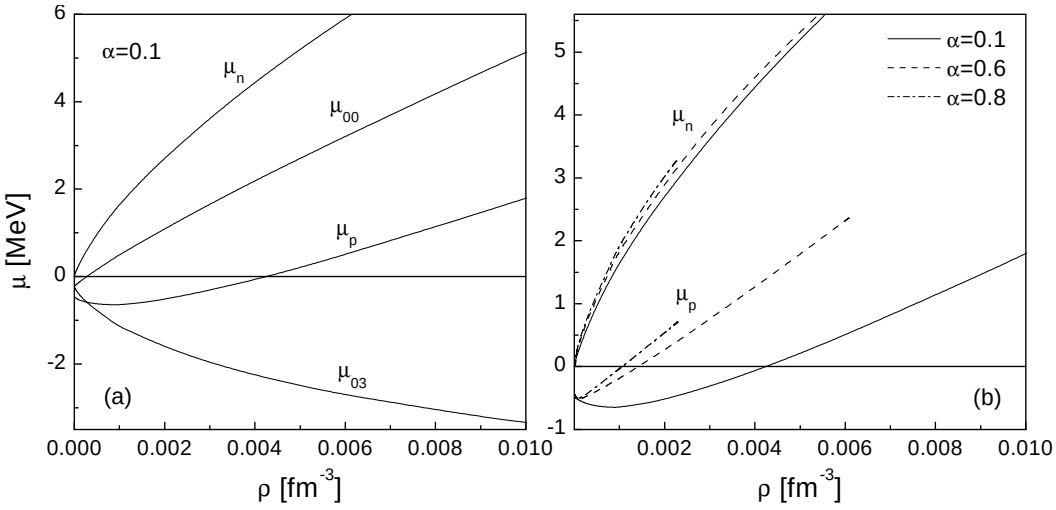

Figure 3. Various chemical potentials as functions of density at zero temperature and different asymmetries: (a) $\mu_{00}, \mu_{03}, \mu_p, \mu_n$; (b) μ_p (lower curves), μ_n (upper curves).

weak coupling regime ($\mu_{00} \gg \Delta$) with increasing asymmetry the width of this domain also increases, that results in considerable reduction of the energy gap magnitude, until it completely vanishes. However, when the chemical potential passes through zero and becomes negative, only part of the window participates in suppressing the energy gap, with the right end of the blocking interval going to zero at $\varrho \to 0$.

In Fig. 3(a) it is shown zero temperature behavior of chemical potentials $\mu_{00}, \mu_{03}, \mu_p, \mu_n$ as functions of density at very low densities of nuclear matter and finite isospin asymmetry. One can see, that at some critical density the chemical potential μ_{00} changes its sign and np Cooper pairs smoothly go over into deuteron bound states. The asymptotic behavior of chemical potentials at $\varrho \to 0$ is $\mu_{00}, \mu_{03} \to -\varepsilon_b/2$, and, hence, $\mu_p \to -\varepsilon_b$, $\mu_n \to 0$. This asymptotic behavior does not depend on isospin asymmetry, that is confirmed by the results of numerical calculations, shown in Fig. 3(b), where the density dependence of chemical potentials μ_p, μ_n is depicted for different values of the asymmetry parameter.

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